Intl J Engg Sci Adv Research 2015 Sep;1(3):27-32

Low Frequency Image Compression via Non-separable Discrete Fractional Fourier Transform

Kritika Mittal
Electronics & Communication Department
Thapar University, Patiala,India
e-mail:kritikamittal1991@gmail.com

***Abstract*—The main idea behind image compression is to reduce the bandwidth for transmission and required space for storage. Thus, image compression is of great importance. In this paper we present the low frequency image compression technique using non-separable discrete fractional Fourier transform (NSDFrFT). Numerical simulation results suggested that image compression method using NSDFrFT as transform technique gives better performance for low frequency images when compression ratio is high. Different image quality measurement methods such as gradient magnitude similarity deviation (GMSD), mean structural similarity index measure (MSSIM), mean squared error (MSE) and peak signal to noise ratio (PSNR) are used to determine the performance.**

Keywords*—*image compression;NSDFrFT; GMSD; MSSIM; MSE; PSNR; DFrFT

I. INTRODUCTION

Storage space required, the transmission bandwidth and the retention of quality of image are important issues to be dealt by image compression [1]-[3].The pixels in image are correlated to each-other and the amount of correlated pixels in an image is termed as redundancy. Moreover, there is some data in image which cannot be noted by human eyes. Thus, such a data is irrelevant [4]. Both redundancy and irrelevancy made image compression possible and effective. A continuous image is sampled and quantized to get a digital image. But such a procedure results in big image which requires large storage space and samples to represent energy. Storage space is a limitation and adding extra storing device is not a solution to the storage problem. So, to counter such problems many image compression algorithms are given [5].

Modest compression is achieved when lossless image compression is done.As no information is lost in the process, thus result is a compressed image identical to the original image. However, in lossy image compression, significant compression can be achieved by discarding redundant data. But the quality of image is degraded due to blocking artifacts [6]. Apart from this image compression methods can be categorized as predictive coding and transform coding. Based on past known values, future values are predicted by predictive coding while transform coding converts the domain of image from spatial to frequency. JPEG 2000 compression coding [7], discrete Fourier transform (DFT), discrete cosine transform (DCT) [8],DFrFT [9], discrete fractional cosine transform (DFrCT) [10], wavelet domain [11], NSDFrFT [12] are some of the image compression algorithms.
The paper discusses the usefulness of the image compression using NSDFrFT algorithm for different types of images, i.e. low frequency images, medium frequency images and high frequency images. In section II,preliminaries of NSDFrFT, DFrFT, GMSD, SSIM, PSNR and MSE are discussed. Section III comprises of the methodology implemented and section IV discusses results with conclusion in section V.

II. PRELIMINARIES

A. *NSDFrFT*

V. Namias [13] first gave the definition of fractional Fourier transform (FrFT), a generalization of Fourier transform (FT). The definition of FrFT for different signals such as one-dimension and multi-dimension, aperiodic and periodic, discrete and continuous was given by Cariolario*et al.*[14].Technology advancement resulted McClellan *et al.*[15] give discrete fractional Fourier transform (DFrFT) as usage of computer and other digital devices increased. The first definition of DFrFT available for calculation purposes was given by S.C.Pei*et al.* [16]. Utilizing non-separable linear canonical transform (NSLCT) definition[17]-[18], definition for FrFT was given by Ozaktas*et al.* [19] and L.B.Almeida [20].

The definition of DFrFT is given as

$$G^{a_1,a_2}[v(x,y)](x,y)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{a_1,a_2}(x,y,x',y')\ v(x',y')dx'dy' \quad (1)$$

where

$$S_{a_1,a_2}(x,y,x',y') = S_{a_1}(x,x')S_{a_2}(y,y') \quad (2)$$

The NSDFrFT, a generalized case of DFrFT was given by A. Sahin*et al.* [21].NSDFrFT uses the concept of interpolation and use of bilinear interpolation was suggested by Sahin. The definition of NSDFrFT has four parameters a_1, a_2, φ_1 and φ_2.

When φ_1 and φ_2 are zero then NSDFrFT converts into DFrFT. Thus NSDFrFT is as follows

$$G_{\varphi_1,\varphi_2}^{a_1,a_2}[v(x,y)] = G^{a_1,a_2}[v(i,j)] \quad (3)$$

where

i=$(cos\varphi_1 x + sin\varphi_1 y)/\cos(\varphi_1 - \varphi_2)$ and

j= $(-sin\varphi_2 x + cos\varphi_2 y)/\cos(\varphi_1 - \varphi_2)$(4)

In-order words x-axis is rotated by φ_1 and y-axis is rotated by φ_2.

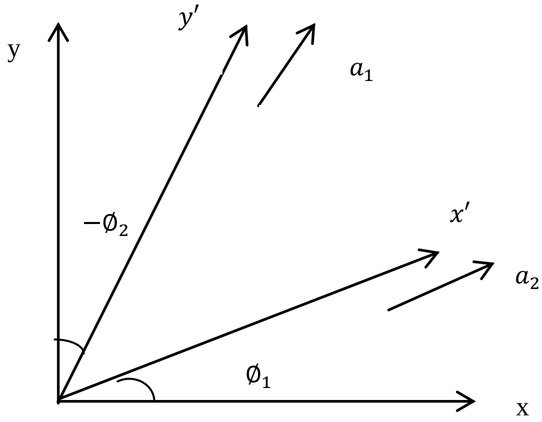


Fig.1. Time-Frequency plane rotation for NSDFrFT

Interpolation plays an important in the definition of NSDFrFT. Thus to enhance the performance of NSDFrFT, use of bicubic interpolation has been suggested instead of bilinear interpolation as suggested by Sahin.

B. Image Quality Parameters

The comparison between the original image and the resultant image of a process based upon an algorithm is done by Objective Quality Metrics (OQM). OQM tells about extend of distortion present in the resultant image when compared with original image. Various OQM methods are PSNR, MSE [22], GMSD [23], MSSIM [24].

The formula for MSE is given a

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j) - K(i,j)]^2 \quad (5)$$

where I and K are the images to be compared.

The formula for PSNR is given as-

$$PSNR = 10.log_{10}\left(\frac{MAX_I^2}{MSE}\right) \quad (6)$$

where MAX_I is the maximum pixel value.

The standard deviation of the GMS map results into the final image quality score known as GMSD [23]. The formula to compute GMSD is given as

$$GMSD = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(GMS(i) - GMSM)^2} \quad (7)$$

where

GMSM is the average value of GMS map forming the resultant final image quality score and GMS is the GMS map at location i. The higher the GMSD score, the more is the distortion in the image.

The mathematical representation of SSIM index is as follows

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (8)$$

where μ_x and μ_y are the mean intensity. σ_x and σ_y are the contrast. C_1 and C_2 are the constants [24].

III. IMAGE COMPRESSION ALGORITHM

The basic concept adopted is similar to that of the JPEG 2000 image compression algorithm [7]. The image intended to be compressed is rotated by φ_1 and φ_2 in x and y direction respectively [21]. Mapping $f(x,y)$ to $f[(cos\emptyset_1 x + sin\emptyset_1 y)/\cos(\emptyset_1 - \emptyset_2), (-sin\emptyset_2 x + cos\emptyset_2 y)/\cos(\emptyset_1 - \emptyset_2)]$ is equivalent to this rotation. Thus, the mapping is achieved by Bicubic interpolation [12]. Substituting

$$i' = (xcos\emptyset_1 + ysin\emptyset_1)/\cos(\emptyset_1 - \emptyset_2)$$

and $j' = (-xsin\emptyset_2 + ycos\emptyset_2)/\cos(\emptyset_1 - \emptyset_2)$ (9)

in the definition of Bicubic interpolation given as [25]

$$f[i',j'] = \sum_{l=-1}^{2}\sum_{m=-1}^{2} f(i+l, j+m)K(l-dx)K(dy-m)$$

where

$$K(x) = \frac{1}{6}[J(x+2)^3 - 4J(x+1)^3 + 6J(x)^3 - 4J(x-1)^3 \text{ and}$$

$$J(x) = \begin{cases} x & x > 0 \\ 0 & x \le 0 \end{cases} \quad (10)$$

In block based image compression algorithm, the image is divided into blocks or sub-images of $4 \times 4, 8 \times 8, 16 \times 16, 32 \times 32$ and many more. The interpolated image is processed block-wise with the block size of 8×8. To reduce the correlation between the pixels of the test image, transform coding is done. On each of this sub-image eq. (3) and (4) is applied. Thus, the target image converts to frequency domain from spatial domain. Less frequency coefficients are required to represent the energy of image as compared to spatial coefficients (pixels).

Quantization of the frequency coefficient to eliminate the irrelevant information from the image is done. Compression ratio is the deciding factor for quantization coefficientwhich is given as

$$CR = \frac{compressed\ image\ bits - original\ image\ bits}{original\ image\ bits} \quad (11)$$

IV. SIMULATION RESULTS

The image compression performance has been compiled for images of three types: High frequency images (Baboon, Grass), Medium frequency images (Barbara, House) and Low frequency images (Pepper, Boat) [26]. For 70%, 50%, 30% and 10% compression, values of PSNR, MSE, MSSIM and GMSD are summarized for optimized values of NSDFrFT using Bicubic interpolation and DFrFT as a transform technique. TABLE I summarize the image quality parameters (IQM) for differentcompression percentages.

From the tabloids it can be concluded that MSE and PSNR for all types of images be it high frequency image or low frequency image or medium frequency images, are better for NSDFrFT in

The encoding process in the reverse order is applied at the decoding end as shown in Fig 3. However, quantization is irreversible, thus in decoding inverse NSDFrFT is applied and sub-images are merged to get reconstructed image.

comparison to DFrFT for high compression percentages [27]-[28]. The image is said to be of high quality when GMSD score is said to be less from that of the compared image. While in case of MSSIM, index should be high in comparison of compared image. From above facts,the low frequency images are of high quality for NSDFrFT than DFrFT. At high compression percentage the extend of distortion in the reconstructed image is high,butNSDFrFT involves interpolation which reduces the visual distortions by smoothening the edges. But at low compression percentages the impairment is minute thus interpolation causes the effect of bluring.So, it can be concluded that NSDFrFT provided better results for all low frequency images and few medium frequency images but none high frequency images at high compression percentages.

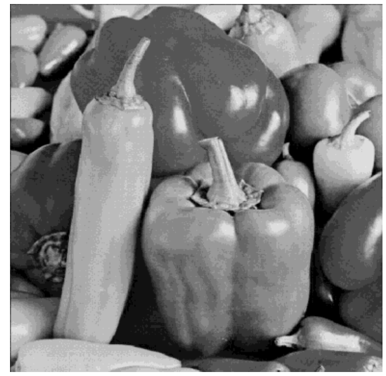
TABLE I. OPTIMIZED IMAGE QUALITY PARAMETERS FOR DIFFERENT TYPES OF IMAGES AT DIFFERENT COMPRESSION PERCENTAGES

Compression Percentages	**Type of Image**		**MSE**		**PSNR**		**MSSIM**		**GMSD**	
			NSDFrFT	**DFrFT**	**NSDFrFT**	**DFrFT**	**NSDFrFT**	**DFrFT**	**NSDFrFT**	**DFrFT**
70	**Low Frequency Image**	**Pepper**	**13.9341**	20.1363	**36.6900**	35.0910	**0.9827**	0.9757	**0.0764**	1.3745
		Boat	**14.9333**	19.0190	**36.3892**	34.7561	**0.9807**	0.8919	**0.1644**	1.5900
	Medium Frequency Image	**Barbara**	**26.7606**	28.1363	**35.0811**	33.4910	0.9800	**0.9830**	**0.0801**	0.9384
		House	**2.4457**	10.1346	**44.2467**	38.0727	0.9891	**0.9930**	0.1112	**0.0272**
	High Frequency Image	**Baboon**	**2.9424**	5.0884	**43.4437**	41.0650	0.9871	**0.9915**	0.0488	**0.0095**
		Grass	**77.6940**	190.013	**29.2280**	26.2799	0.9674	**0.9713**	0.0629	**0.0565**
50	**Low Frequency Image**	**Pepper**	**7.7921**	14.4629	**39.2142**	36.5283	**0.9925**	0.9925	**0.0698**	1.7222
		Boat	**5.8015**	94.9666	**40.4954**	28.3551	**0.9901**	0.9850	**0.1516**	2.9954
	Medium Frequency Image	**Barbara**	**14.4629**	20.1822	**40.1371**	36.5283	0.9893	**0.9941**	**0.0690**	0.1656
		House	**1.9454**	4.0101	**45.1699**	40.7046	0.9967	**0.9987**	**0.1033**	0.2441
	High Frequency Image	**Baboon**	**2.1722**	2.9424	**44.7618**	43.7051	0.9968	**0.9990**	**0.0381**	0.0809
		Grass	**18.2413**	24.7304	**35.5202**	28.3659	0.9885	**0.9970**	0.0529	**0.0220**
30	**Low Frequency Image**	**Pepper**	3.1950	**2.346**	43.0862	**44.42**	**0.9977**	0.9966	**0.0372**	0.0657
		Boat	**2.0525**	3.860	**48.3115**	46.45	**0.9961**	0.9850	**0.0725**	2.5980
	Medium Frequency Image	**Barbara**	**3.0040**	3.046	**43.5115**	42.29	0.9913	**0.9995**	0.0322	**0.0127**
		House	**2.0823**	2.445	**44.9453**	44.24	0.9974	**0.9995**	0.1261	**0.0218**
	High Frequency Image	**Baboon**	1.3538	**0.572**	46.8154	**50.55**	0.9988	**0.9998**	0.0361	**0.0105**
		Grass	11.3192	**7.631**	39.0760	**39.30**	0.9948	**0.9992**	0.0227	**0.0045**
	Low Frequency Image	**Pepper**	2.5383	**2.006**	44.0853	**52.64**	**0.9998**	0.9982	**0.0144**	0.0529
		Boat	1.4633	**0.496**	**52.7562**	57.71	**0.9968**	0.9102	**0.0715**	0.0715

10	**Medium Frequency Image**	**Barbara**	0.3105	**0.206**	48.8340	**54.64**	0.9928	**1.0000**	0.0168	**0.0096**
		House	2.0610	**0.948**	44.9900	**48.35**	0.9976	**1.0000**	0.1251	**0.0023**
	High Frequency Image	**Baboon**	1.1266	**0.246**	47.6130	**62.64**	0.9990	**1.0000**	0.0124	**0.0016**
		Grass	1.2698	**0.306**	46.2479	**53.26**	0.9955	**0.9999**	0.0213	**0.0006**


(a) Original Peppers (Low Frequency Image)


(b) Processed Image via NSDFrFT


(c)Processed ImageviaDFrFT

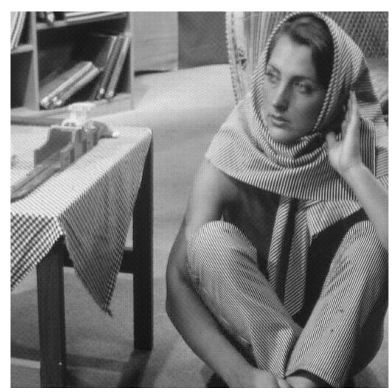

(d) zoomed portion of (a)

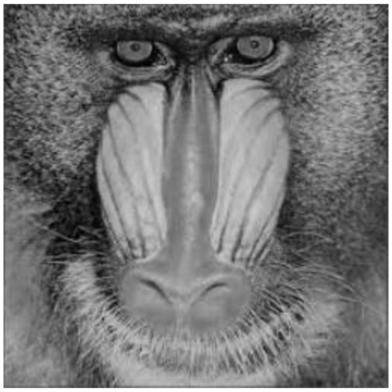

(e)zoomed portion of (b)


(f) zoomed portion of (c)

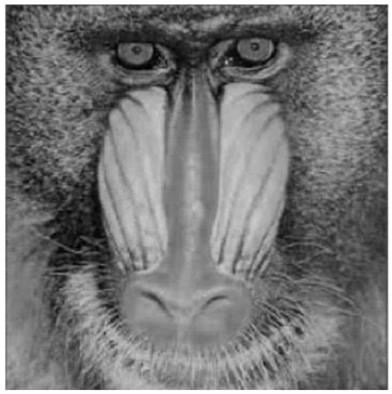

(g) Barbara (Medium Frequency Image)


(h) Processed Image via NSDFrFT

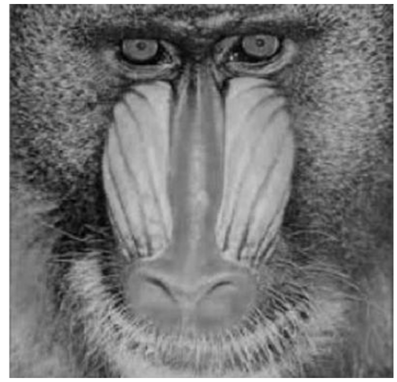

(i) Processed Image via DFrFT



(j) Baboon (High Frequency Image)



(k) Processed Image via NSDFrFT



(l) Processed Image via DFrFT

Fig.2. Original images of three types and their processed images viaNSDFrFT and DFrFT for 70% compression.

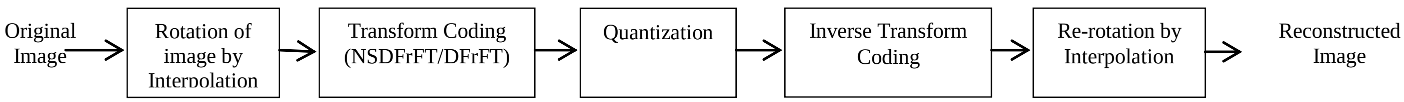


Fig.3. Complete procedure of compression and decompression using NSDFrFT and DFrFT.

V. Conclusion

Image compression performs better in transform domain than in spatial domain when NSDFrFT and DFrFTis used as transform coding. The image has been compressed for higher compression ratios and different image quality measures are used to estimate the quality of processed image via NSDFrFT and DFrFT. The IQMs PSNR, MSE, MSSIM and GMSD suggested that the processed image using NSDFrFT is of higher quality than the processed image via DFrFT at higher compression percentages. Images when classified into high, medium and low frequency image than IQM suggested that image compression using NSDFrFT performs better for all low frequency images as GMSD score and SSIM index is good for them. So, in simple words for low frequency images MSE, PSNR, MSSIM and GMSD are better than medium and high frequency images when compressed with NSDFrFT.

References

[1]. K. A. McIntyre, “Dynamic Bandwidth Adaptive Image Compression/ Decompression Scheme,” U.S. Patent 7024045, 2006.

[2]. T. L. B. Yng, B. G. Lee and H. Yoo, “A Low Complexity and Lossless Frame Memory Compression for Display Devices,” *IEEE Trans. Cons. Electron.*, vol. 54, no. 3, pp. 1453-1458, 2008.

[3]. J. Shukla, M. Alwani and A. K. Tiwari, “A Survey on Lossless Image Compression Methods,” in Proc. *Second International Conf. on Computer Engineering and Technology* (ICCET), vol. 6, pp. 136-141, 2010.

[4]. S. Jayaraman, S. Esakkirajan and T. Veerakumar, *Digital Image Processing*, Tata McGraw-Hill Education, 2011.

[5]. R. C. Gonzalez and R. E. Woods, *Digital Image Processing*, 3rd edition, 2008.

[6]. K. S. Thyagarajan, *Still Image and Video Compression with MATLAB*, John Wiley & Sons, Inc. Hoboken, New Jersey, 2011.

[7]. Skodras, C. Christopoulos and T. Ebrahimi, “The JPEG 2000 still Image Compression Standard,” *IEEE Trans. on Sig. Process*, vol. 18, no.5, pp. 36-58, 2001.

[8]. B. Watson, “Image Compression using the Discrete Cosine Transform,” *Math. Journal*, vol. 4, no. 1, pp. 81-88, 1994.

[9]. N. Jindal and K. Singh, “Image and Video Processing using Discrete Fractional Transforms,” *Sig. Image and Video Process*, DOI: 10.1007/s11760-012-0391-4, 2012.

[10]. K. Singh, “Performance of Discrete Fractional Fourier Transform Classes in Signal Processing Applications,” Ph.D Thesis, Dept. of Elect. Comm. Eng., Thapar Univ., Patiala, India, 2005.

[11]. S. Grgic, M. Grgic and B. Zovko-Cihlar, “Performance Analysis of Image Compression using wavelets,” *IEEE Trans. Ind. Electron.*, vol. 48, no.3, pp. 682-695, 2001.

[12]. K.Mittal and K.Singh, “Image Compression using 2D non-separable Discrete Fractional Fourier Transform”,unpublished.

[13]. Namias, “the fractional order fourier transform and its applications to quantum mechanics,” *j. Inst. Math applications*, vol. 25, pp. 241-265, 1980.

[14]. G. Cariolario, T. Ersrghe, P. Kraniauskas, and N. Laurenti, “A unified framework for the fractional fourier transform,” *IEEE Trans. on Sig. Process*, vol. 46, pp. 3206-3212, 1998.

[15]. B. Santhanam, and J. H. McClellan, “The DFRFT-A Rotation in Time Frequency Space,” *IEEE Trans. on Sig. Process.*, no. 5, pp. 921–923, 1995.

[16]. S. C. Pei, and J. J. Ding, “Closed-Form Discrete fractional and Affine Fourier Transforms,” *IEEE Trans. on Sig. Process*, vol.48, pp.1338-1353, 2000.

[17]. Koc, H.M. Ozaktas and L. Hesselink, “Fast and accurate computation of two-dimensional non-separable quadratic-phase integrals,” *Journal of the Optical Society of America*, vol.27, pp.1288–1302, 2010.

[18]. J.J. Ding and S.C. Pei, "Heisenberg's uncertainty principles for the 2-D nonseparable linear canonical transforms," *Elsevier on Sig. Process,* vol.93, pp. 1027-1043, 2013.

[19]. H. M Ozaktas, Z. Zalevsky and M. A. Kutay, "The fractional Fourier Transform with Applications in Optics and Signal processing," *John Wiley & Sons Ltd.,* New York, 2000.

[20]. L.B. Almeida, "The fractional Fourier transform and time–frequency representations," *IEEE Trans. on Sig. Process,* vol.42, pp. 3084-3091, 1994.

[21]. A.Sahin, M.A.Kutay and H.M. Ozaktus, "Nonseparable two-dimensional fractional Fourier transform,"*Journal of the Optical Society of Americaon Applied Optics*, vol. 37, no.23, pp. 5444- 5453, 1998.

[22]. I.E.G. Richardson, *H.264and MPEG-4 Video Compression,* John Wiley & Sons, Inc. West Sussex, England, 2003.

[23]. W.Xue, L.Zhang, X.Mou and A.C. Bovik, "Gradeint Magnitude Similarity Deviation: An Highly Efficient Perceptual Image Quality Index", *IEEE Tranc.on Image Process.,vol.23, no-2, pp. 684-695, 2014.*

[24]. Z.Whang, A.C. Bovik, H.R.Sheikh and E.P.Simoncelli, "Image Quality Assessment: From Error Visibility to Structural Similarity",*IEEETranc. On Image Process.*, vol.13, no.4, pp. 600-612, 2004.

[25]. J.A. Parker, R.V. Kenyon and D.E. Troxel, "Comparison of Interpolating Methods for Image Resampling," *IEEE Trans. on Med. Imag*, vol MI-2, no.1, pp. 31-39, 1983.

[26]. S. Rout, "Orthogonal vs. Biorthogonal Wavelets for Image Compression", M.S.Thesis, Dept of Elec. Engg, Virginia Tech, Blacksburg, Virginia, 2003.

[27]. N. Jindal and K. Singh, "Joint Image Compression-Encryption Using Discrete Fractional Transforms," *The Imag.Sci. Journal*, DOI 10.1179/1743131X13Y.0000000062, 2013.

[28]. N. Jindal and K. Singh, "Image Retrieval Algorithm based on Discrete Fractional Transforms", Journal of Electrical Engineering, vol. 64, no. 4, pp. 250-255, 2013.