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Modal Analysis and Cutoff Condition of a Doubly Clad Cardioidic Waveguide

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***Abstract*— In this paper, we choose a simple and practical analytical method based on scalar wave approximation to study the modal behaviour, cutoff condition of a cardioidic doubly clad optical waveguide. The proposed waveguide is composed of three portions: the core, inner cladding, and outer cladding. It is assumed that the core has the largest refractive index of the three, and the outer cladding index is the next largest. We take the appropriate orthogonal coordinates for the proposed structure and impose the boundary conditions under the weak guidance approximation to get the modal eigen value equation. Using this equation we obtain numerical results in the form of dispersion curves and cutoff frequencies. An attempt has been made to estimate how the modal behaviour and cutoff conditions change as singly clad circular fiber to the doubly clad circular fiber.**

Keywords— waveguide, semiconductors lasers

I. Introduction

Light wave communication systems, using optical fibers as the communication medium, are already established as the important configurations for transmission networks [1]. The study of electromagnetic wave propagation through optical waveguides of non-circular cross-sections presents considerable difficulties and rigorous analytical results are very hard to obtain. Using numerical or partly analytical methods several investigators have also studied slab waveguides with deformations resulting in circular or parabolic curvatures in the transverse sections of the core cladding boundaries [2-4]. Modal analysis of non-circular waveguides has been thoroughly done and appears extensively in the literature [5-10]. Planar, slab and multilayered waveguides are also used in the implementation of a verity of optical devices including semiconductors lasers, modulators, waveguides polarizer's, optical sensors etc. Generally, non-circular waveguides have a guiding region of non-circular cross-section surrounded by a closed or partially closed core cladding interface boundary. One region for the study of such non-circular cross-sections in waveguides is to find out the effect of a distortion in the usual circular cross-section. Such a distortion may not always be of a symmetrical nature. When a circle is distorted into an ellipse, the distortion is symmetrical about to perpendicular diameters. However a circle may also be given distortion when the resulting shape is less symmetrical making the shape symmetrical about only one diameter and ellipsoid with respect to the perpendicular diameter. Finally, the distortion may also be such that the resulting shape is totally lacking in symmetry [11]. Also there are many optical waveguides with double-layer or multilayered structures [12-13]. The basic advantages of these type of waveguide is to have low dispersion over a wide wavelength range [14-15] and also attractive for high –bit-rate light wave communication system [16-17]. Hence in this paper we will explore some of the fundamental properties of a cardioidic –shaped doubly clad optical waveguides with special reference to their use in optical communications system. Theory - The cross section view of the core of proposed waveguide is shown in figure (1) and the index profile of the proposed waveguide is shown in figure (2). We consider the core region has largest refractive index of the three, and the outer cladding index is the next largest. Thereby we have suitably designed alternating claddings of low and high refractive indices. Here we have taken (r,θ) as polar coordinate on the cross sectional plane but we now choose new coordinates (ξ, η, z) suitable for the proposed geometry. The direction of propagation is along z-axis, which is normal to the plane of paper in figure (1). We introduce new coordinate system by assuming that we have an infinite set of cardioids of varying sizes a, now represented by the variable ξ. Next we have another infinite set of orthogonal cardioids of size b, now represented by variable η. The details of this procedure are given in our previous paper [18]. The axial field components can be written as

For core region

$$E_{core} = AJ_{1/4}(2u\xi) \tag{1}$$

for inner cladding region

$$E_{clad\,\mathrm{I}} = BI_{1/4}(2w_1\xi) + CK_{1/4}(2w_1\xi) \tag{2}$$

for outer cladding region

$$E_{clad\,\mathrm{II}} = DK_{1/4}(2w_2\xi) \tag{3}$$

Where

$$u^2 = \omega^2 n_1^2 \mu_1 - \beta^2,\ w_1{}^2 = \beta^2 - \omega^2 \mu_2 n_2^2$$

and $w_2{}^2 = \beta^2 - \omega^2 \mu_3 n_3^2$.

Also A, B, C, and D are unknown constants and n_1, n_2, and n_3 are the permittivity of the core, inner and outer cladding region respectively. Here $\mu_1=\mu_2=\mu_3=\mu_0$ is the permeability of the medium. J_n is the Bessel function for the guiding region I_n and K_n are the modified Bessel function for the inner and outer cladding regions respectively.

Here β is the axial component of propagation vector, ω is the wave frequency, μ_0 is the permeability of non-magnetic medium. The boundary conditions can be written as

$$E_{core}\big|_{\xi=a} = E_{clad\,\mathrm{I}}\big|_{\xi=a} \tag{4}$$

$$E_{clad\,\mathrm{I}}\big|_{\xi=a1} = E_{clad\,\mathrm{II}}\big|_{\xi=a1} \tag{5}$$

$$\left.\frac{dE_{core}}{d\xi}\right|_{\xi=a} = \left.\frac{dE_{clad\,\mathrm{I}}}{d\xi}\right|_{\xi=a} \tag{6}$$

$$\left.\frac{dE_{clad\,\mathrm{I}}}{d\xi}\right|_{\xi=a1} = \left.\frac{dE_{clad\,\mathrm{II}}}{d\xi}\right|_{\xi=a1} \tag{7}$$

Thus we get a set of equations having four unknown constants. The non trivial solution will exist only when the determinant formed by the coefficients of the unknown constants is equal to zero. Hence we have

$$\begin{vmatrix} J_{\frac{1}{4}}(2ua) & -I_{\frac{1}{4}}(2w_1a) & -K_{\frac{1}{4}}(2w_1a) & 0 \\ uJ'_{\frac{1}{4}}(2ua) & -w_1I'_{\frac{1}{4}}(2w_1a) & -w_1K'_{\frac{1}{4}}(2w_1a) & 0 \\ 0 & I_{\frac{1}{4}}(2w_1a1) & K_{\frac{1}{4}}(2w_1a1) & -K_{\frac{1}{4}}(2w_2a1) \\ 0 & w_1I'_{\frac{1}{4}}(2w_1a1) & w_1K'_{\frac{1}{4}}(2w_1a1) & -w_2K'_{\frac{1}{4}}(2w_2a1) \end{vmatrix} = 0$$

eq. (8)

The prime ($^{/}$) of above equation represents differential with respect to the argument. The dimensionless V-parameter is introduced to incorporate the parameters n_1, n_2, n_3, a, $a1$ and k_0 which may possibly have an effect on the propagation.

$$V = k_0 a (n_1^2 - n_2^2)^{\frac{1}{2}} \tag{9}$$

where k_0 is vacuum wavenumber. We define the usual normalized propagation parameter

$$b^{/} = \frac{\beta^2 - k_0^2 n_3^2}{k_0^2(n_1^2 - n_3^2)} \qquad \text{(weakly guidance case)} \tag{10}$$

II. Numerical Computations, Results and Discussion:

The characteristic equation (8) contains all of the information that we can obtain from our modal analysis and it gives the central results of this investigation. We now proceed to some numerical computation in order to have the modal dispersion curves for the proposed waveguide. It is convenient to plot the normalized propagation constant $b' = \dfrac{\frac{\beta^2}{k_0^2} - n_3^2}{n_1^2 - n_3^2}$ against the V-parameter defined by $V = \dfrac{2\pi a}{\lambda_0}(n_1^2 - n_2^2)^{\frac{1}{2}}$. Now we choose the refractive indices $n_1=1.50$, $n_2=1.20$, $n_3=1.30$ for core, inner cladding and outer cladding respectively also an operating wavelength $\lambda_0 = 1.55\mu m$ and various values of dimensional parameter *a* in a regular increasing order. For each value of *a* we obtain the V-parameter and also

TABLE I

Cutoff V value	When inner cladding *a1=0.2*	When inner cladding *a1=0.5*	When inner cladding *a1=1*	When inner cladding *a1=2*
V_1	1.0943	1.0943	1.0941	1.0943
V_2	2.9181	3.6477	3.2829	3.2829
V_3	5.1067	5.8363	5.1067	5.1067
V_4	6.9306	6.9306	6.9306	6.9306
V_5	8.7544	8.7544	8.7544	8.7544
V_6	10.943	10.5783	10.5783	10.5783

compute the values of β from the characteristic equation (8) by graphical method. It means that the left hand side of characteristic equation is plotted against β for the assumed value of *a* and the zero crossing of the graph with the β axis are noted. These values are the solutions of the characteristic equation for the different modes. For example the lowest zero crossing value of β corresponds to the lowest order mode. From these value of β we can compute the values of $b^/$ and then plot the dispersion curves for the different modes. These graphs are shown in figure (3) to figure (6) for these modes. These dispersion curves have the expected shape, which means that the doubly clad cardioidic waveguide does not cause a change in the shape of dispersion curves. The cutoff frequencies obtained from the characteristic equation (8) are shown in table 1.

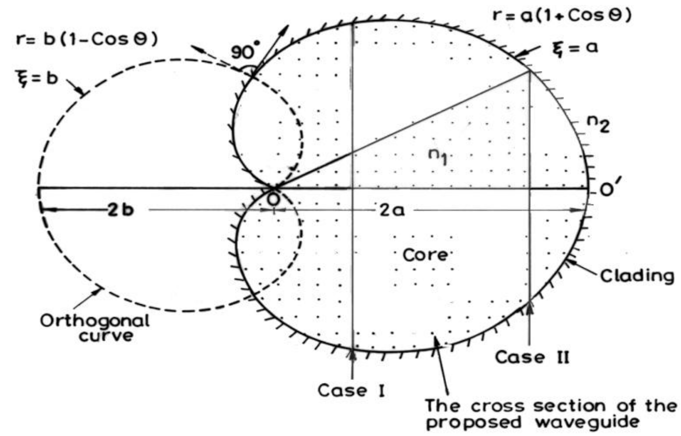

Fig. 1

Fig. 1. The cross section view of the core of proposed doubly clad cardioidic waveguide.

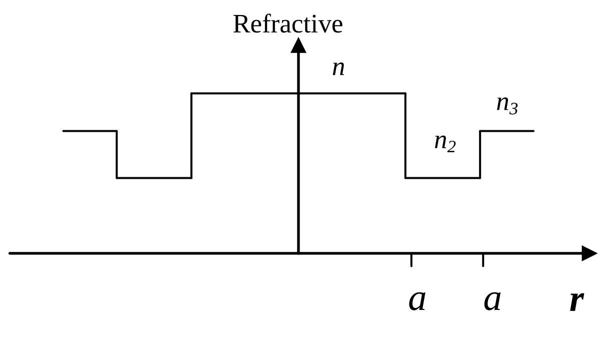

Fig. 2. Index profile of a weakly guiding fiber waveguide.

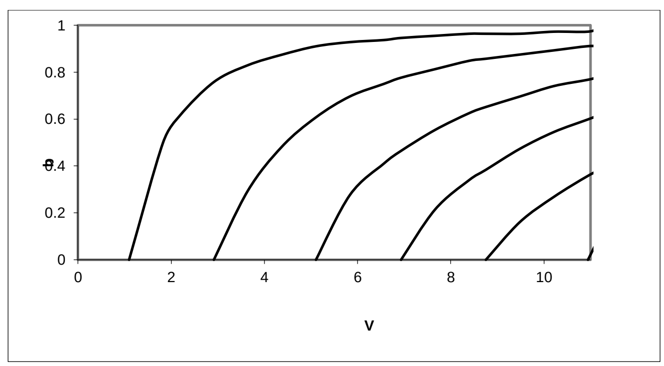

Fig. 3. Dispersion curves ($b^/$ versus V) of a few lowest modes for the proposed waveguide at *a1=0.2*.

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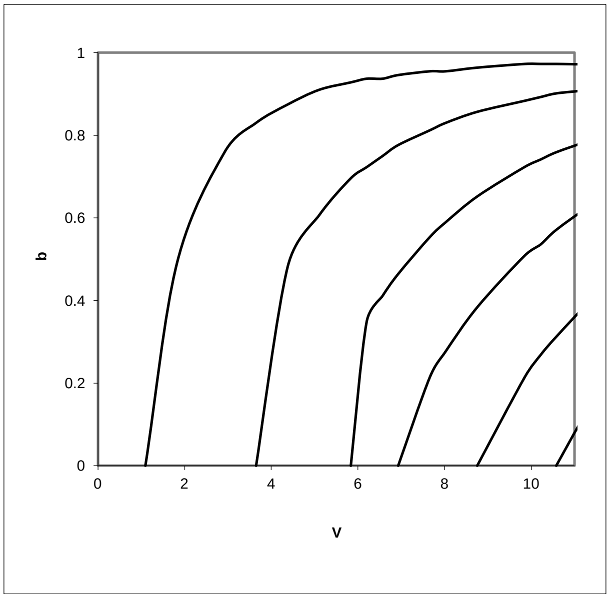

Fig. 4. Dispersion curves ($b^{/}$ versus V) of a few lowest modes for the proposed waveguide at *a1=0.5*.

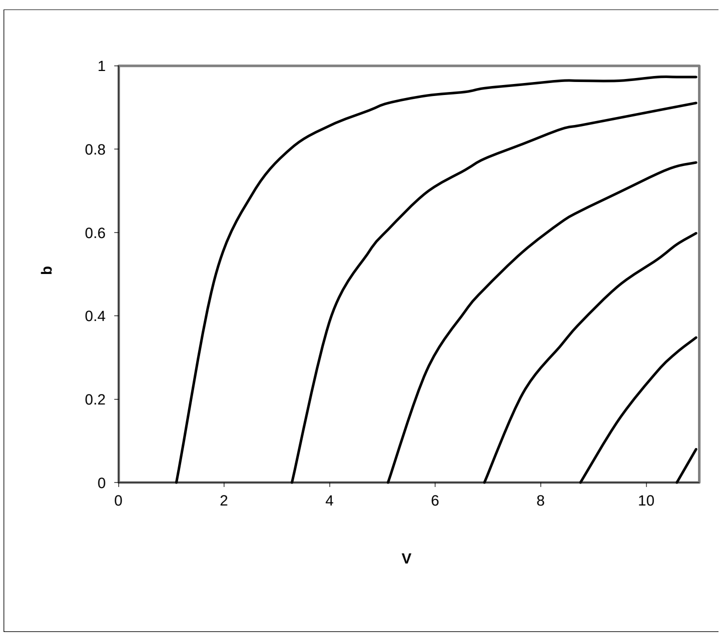

Fig. 5. Dispersion curves ($b^{/}$ versus V) of a few lowest modes for the proposed waveguide at *a1=1*.

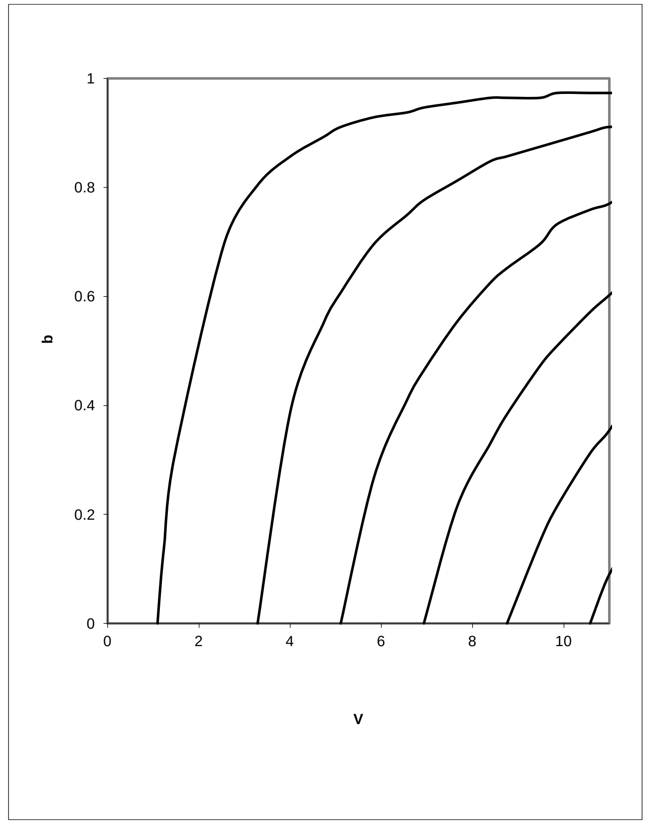

Fig. 6. Dispersion curves ($b^{/}$ versus V) of a few lowest modes for the proposed waveguide at *a1=2*.

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