Zeeman Term Effect on Topological Insulators Thin Film

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Abstract.3D topological insulators have surface states that consist of mass less 2D Dirac fermions. Such relativistic fermions systems are best characterized using the phenomenon of anomalous Rabi oscillation that occurs far from conventional resonance and is unique to these systems. Anomalous Rabi oscillations are absent in two level atoms and conventional semiconductors. On applying external magnetic field perpendicular to the thin film of surface states of TI, Zeeman term induced. It is shown that the Zeeman term has no qualitative effect on the conventional Rabi frequency but its absence makes the anomalous Rabi modes mass less. This observation reinforces that the anomalous Rabi oscillation is sensitive to qualitative changes in the low-energy band structure rather than the conventional Rabi oscillation and compared these results with grapheme.

I. INTRODUCTION

Topological insulators (TI) are peculiar quantum systems [1], which have an insulating gap in the bulk due to the presence of spin-orbit coupling, and possess intrinsic Dirac metallic states at the surface. The first nontrivial topological state was observed in the phenomenon of quantum Hall Effect, where two-dimensional electron system subject to a perpendicular magnetic field was seen to possess a quantized Hall conductivity. There is the presence of the conductive quantized channel on the edges of the system was associated with the nontrivial topological class of its bulk [2]. In 2005, C. L. Kane and E. J. Mele discovered a new model to obtain a non-trivial topological state without breaking time-reversal symmetry without an external B-filed, based on the spin-orbit coupling applied to the graphene model Hamiltonian [3]. Firstly, this model shows the phenomenon of quantum Spin Hall effect, also known as a 2D topological insulator and opens the way for the investigation of a new topological class, called Z_2 topological insulators. This concept is expanded further in the 3D topological insulator, which are 3D insulating systems which have 2D exotic metallic states at all interfaces with vacuum or another dielectric. Such topological edge states are protected by time-reversal symmetry and due to spin-orbit coupling, they are spin-momentum locked, which means that the direction of their motion uniquely determines their spin polarization and vice versa.

A periodic exchange of energy between the light field and two level systemsis known as Rabi oscillations [4], and the frequency is determined by the intensity of external applied field. The Rabi frequency is defined as

$$\omega_R = \frac{|\boldsymbol{p} \cdot \boldsymbol{E}|}{\hbar}$$

Where **p** is dipole moment and **E** is the external applied field. Conventional Rabi oscillations are studied using the rotatingwave approximation (RWA) [5]. This is an approximation, where rapidly oscillating terms of the effective Hamiltonianare removed. This approximation is valid near the resonance i.e. when the incident frequency of the light field nearlyequal to transition frequency of the two level systems. In the off-resonance case, a new kind of Rabi oscillation is seen,in topological insulator. To study this, we employ an approximation known as asymptotic rotating wave approximation (ARWA) [6–11] (otherwise known as Fouquet approximation [12, 13]). The dephasing of anomalous Rabi oscillations in monolayer graphene can also be seen by including the electron-phonon interaction [14]. In this article, the effect of Zeeman term on anomalous Rabi oscillation is shown and how this is sensitive to qualitative changes in the low-energy band structure rather than the conventional Rabi oscillation.

II. ANOMALOUS RABI OSCILLATIONS IN TOPOLOGICAL INSULATOR

The two-dimensional Hamiltonian of an ultrathin TI film for these Dirac fermions with hybridization due to quantum tunneling between the top and bottom surfaces of the bulk TI is given by [15,16]

$$H = \int d^2\boldsymbol{r}\, \psi^{\dagger}(\boldsymbol{r}) \left[v_F \tau_z (\hat{z} \times \boldsymbol{\sigma}) \cdot \left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right) + \boldsymbol{\Delta} \cdot \boldsymbol{\sigma} + \Delta_t \tau_x \right] \psi(\boldsymbol{r})$$

Here is the hybridization matrix element between the states of the top and bottom surfaces of the TI. Its value depends on the thickness of the 3D TI. The eigenvalues of which are stand for symmetric and anti-symmetric surface states respectively and the Zeeman energy is is the projection of $\boldsymbol{\Delta} = \frac{1}{2}\, g\, \mu_B\, \boldsymbol{B}(t) = \frac{1}{2}\boldsymbol{\Delta}_0 e^{i\omega t} + \frac{1}{2}\boldsymbol{\Delta}_0^* e^-$
On the μ_B is the Bohr magnetron. We also write the vector potential as $\boldsymbol{A}(t) = \frac{1}{2}\mathbf{A}_0 e^{i\omega t} + \frac{1}{2}\boldsymbol{A}_0^* e^-$ so that $\boldsymbol{\Delta}_0 = \frac{1}{2}\, g\, \mu_B(-i\, \boldsymbol{k} \times$ A_O). Without loss of generality we may assume that the surfaces of the TI are in the xy plane.

For solving energy eigen value equation $H\,\psi =$ require the use of approximations. If driving frequency is closed to one of the excitation or a resonant frequency of the system, rotating wave approximation (RWA) is used, described in many text book [7]. In case of off-resonant case, we apply a new method has been developed by our group, known as asymptotic rotating wave approximation (ARWA) [8–13]. In off-resonant case, the external driving frequency is always much larger than the Rabi frequency and the resonant frequency for the creation of particle hole pairs namely $2v_F|\boldsymbol{p}|, (\omega \gg \omega_R$ and $\omega \gg 2v_F|$. This condition is quite dissimilar from the condition use in rotating wave approximation (RWA) namely $|\omega - 2v_F|\boldsymbol{p}| \lessdot$. Firstly, we break H and $\psi($ (p, t) in terms of slow and fast parts $H = H_0 + e^{-i\omega t}\, V_+ + e^{i\omega t}$ and $\psi = \psi_0 + e^{-i\omega t}\, \psi_+ + e^{i\omega t}$. Here, H_0 and $\psi($ represents slow parts, on other hand ; and ; are the (coefficients of) fast parts of the full Hamiltonian and wave function, respectively. Itis considered that external driving frequency is much larger compare to band gap. After putting these values in to equation $H\psi =$ and leaving higher harmonics i.e. order of . Finally, we will get energy eigen value equation interm of slow part only $H_{eff}\psi_0 = E$. We defined, $H_{eff} = \left(H_0 + \frac{[V_-]}{\omega}\right)$ The eigen values of H_{eff} are known as *anomalous Rabi frequency*.

III. RESULT AND DISCUSSION

We first focus on the case of linear polarization. We have seen in earlier works that anomalous Rabi oscillations are absent in two dimensional graphene when radiation is linearly polarized [8, 10]. Here too the situation is similar except that the hybridization induces an intrinsic frequency scale which cannot be thought of Rabi oscillation. However, when

(i). EM Wave propagation **k** in z direction, in x direction, in y Direction then

$$\Omega_{ARWA} = \sqrt{\Delta_t^2 + v_F^2(p_x^2 + p_y^2)}$$

(ii). EM wave propagation **k** in x direction, A_0 in y Direction, Δ_0 in z direction. Here $\omega_R' = \frac{e\, v_F}{c}|A(0)|$ is the scale of the conventional Rabi frequency and $|A(0)|$ is the magnitude of the vector potential, then

$$\Omega_{ARWA} = \sqrt{\left(v_F p_x - \frac{g\,\mu_B \omega_R'^2}{2e\,v_F}\right)^2 + \Delta_t^2 + v_F^2 p_y^2} \quad (2)$$

The anisotropic dependence of the frequency on the wave vector due to the coupling of the spin with the magnetic field is clearly seen. We may also note that in a system with vanishing hybridization (thick samples) and the region $v_F p_y := 0$ this frequency has a mass less character i.e., the dispersion relation with the remaining component is linear and the frequency vanishes at $v_F p_x = \frac{g\,\mu_B \omega_R'^2}{2e\,v_F} \equiv \epsilon_0$ which is the Zeeman energy scale, rather than at $v_F p_x := 0$ To be consistent, we have to ensure that this point is in the ARWA regime. This is possible only if $\frac{g\,\mu_B \omega_R'^2}{2e\,v_F} \ll \omega$

Next, we focus on the case of circular polarization and grazing incidence (direction of propagation radiation is taken to be x-direction and the surfaces are in the x y plane). In graphene, in this case, no anomalous Rabi oscillations are seen [10]. However, in the present example, we do see anomalous Rabi oscillations with an anisotropic dependence on the wave vector due to the coupling of spin with the magnetic field (EM field).

(iii). EM wave propagation in x-direction (grazing incidence) with beam circularly polarized in the y z plane. In this case, we see two branches in the anomalous Rabi frequency.

$$\Omega_{ARWA} = \frac{1}{\omega}\left[\pm\frac{2(\epsilon_0\,\omega)^2}{\omega_R'^2}\sqrt{\gamma^2 + \delta^2\frac{p_y^2}{p^2} + (\epsilon_0\,\omega)^2\left(1 + \left(\frac{\epsilon_0\,\omega}{\omega_R'^2}\right)^2\right)} + 2\,\delta\,(\epsilon_0\,\omega)\frac{p_x}{p} + \gamma^2 + \delta^2\right]^{\frac{1}{2}}$$

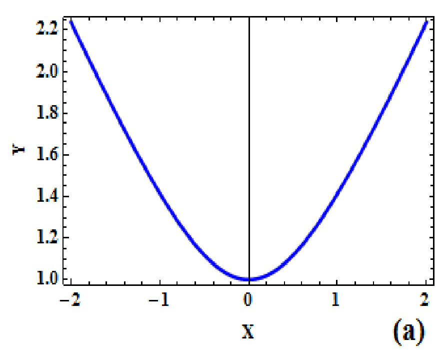
The form of Eq. (3) is quite illuminating. From case (ii) we know that $\epsilon_0 \sim v_F p_x \text{ - } 0$ in the limit $\omega - \infty$ such the $0 < \epsilon_0\omega < \infty$ Of course in ARWA: $0 < \gamma, \delta < \infty$ As $\omega - \infty$. Hence the form of Eq. (3) is consistent with ARWA which demands $\Omega_{ARWA} \sim$ constant/ω as $\omega -. \infty$ In addition, the anisotropic dependence of this Rabi frequency on the wave vector is also apparent. It is interesting to consider the special case of vanishing hybridization $\gamma := 0$ and $v_F p_y := 0$. In this case

$$\Omega_{ARWA} = \sqrt{\left(\frac{z_0^2}{\omega}\right)^2 + (v_F p_x + \epsilon)^2} \quad (4)$$

Where, $z_0 = \epsilon_0\omega/\omega_r$ In this case we see that there is a non vanishing minimum anomalous Rabi frequency viz. Z_0^2/ω.. This minimum is not at the Dirac point $p_x = p_y := 0$ but shifted relative to this point $v_F p_x = -\epsilon_0$ this is a feature we also saw in Weyl materials but not in graphene. We may think of this feature as a kind of intrinsic Bloch-Sie gert shift [6] not brought about by frequency doubling or the inclusion of counter rotating terms, but by the Zeeman term. In general however, whenever hybridization may be ignored,

$$\Omega_{ARWA} = \frac{\sqrt{\left(\omega\, v_F p_y + \frac{(\epsilon_0\,\omega)^2}{\omega_R'^2}\right)^2 + \omega^2 (v_F p_x + \epsilon_0)^2}}{\omega} \quad (5)$$

A plot of Eq.(5) shows the massless character of the Rabi modes on the surface of topological insulators in the special case of grazing incidence and circularly polarized light in the vicinity of the point $(v_F p_x, v_F p_y) = \left(-\epsilon_0, -\frac{(\epsilon_0\,\omega)}{\omega\,\omega_R'^2}\right)$



(a)

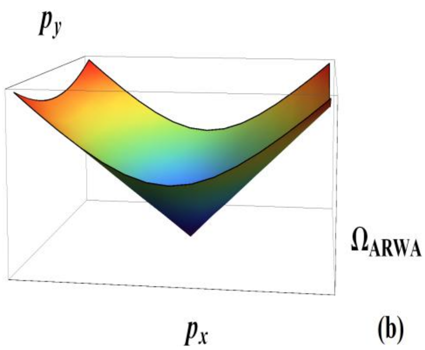


FIGURE 1.(Color online) Plot **(a)** demonstrates (Eq.(4)) the anomalous Rabi frequency versus the x-component of the wave vector when the y-component is made zero. Even though the plot amplifies the massive character of this Rabi mode, it is nearly mass less and the small non-zero minimum value is due to the Zeeman energy. The labels in this plot are $Y = \frac{\Omega_{ARW}}{5.45\times10^{-}}$, $\frac{v_F p_x}{\omega} = 5.45\times10^{-12}(X - 1652.0)$ Plot **(b)** demonstrates 3D plot on the bottom (Eq.(5)) depicts the mass less Dirac-like cone of the anomalous Rabi frequency at a shifted point in the wave vector space determined by the Zeeman energy scale.

(iv). In the case of circular polarization and normal incidence, the anomalous Rabi frequency $\sqrt{\left(\frac{\omega_{R,*}^{2}}{\omega}\right)^2 + (v_F p)^2}$ is isotropic and closely resembles the form seen in graphene [9], where $\omega_{R,*} = \sqrt{\omega_R'^2 + \pm\Delta_t\omega + \left(\frac{\epsilon_0}{\omega}.\right)^2}$

As a comparison we now wish to study conventional Rabi oscillations in this system as well. As usual, the interesting phenomena take place when the hybridization is vanishingly small $(\Delta_t = 0)$ It so happens that this limit is mathematically simpler as well. In the case studied earlier viz. grazing incidence and circular polarization, and when in addition we set $v_F p_y$;=0 the anomalous Rabi frequency is given by Eq.(4) which shows that the small mass in these modes is purely due to the Zeeman term and it is mass less when the Zeeman term is absent. On the other hand, in this case, the conventional Rabi frequency is found to be,

$$\Omega_{RWA} = \sqrt{\left(\omega_R' \pm \frac{\epsilon_0\omega}{\omega_R'}\right)^2 + \delta'^2} \quad (6)$$

Where $\delta' = \omega - 2\, v_F p$ is the detuning? This Eq. (6) shows that the modes are massive even in the absence of the Zeeman term. This observation reinforces the point we have making in our earlier works [6–11] as well, viz. it is the anomalous Rabi oscillation that is sensitive to qualitative changes in the low-energy band structure rather than the conventional Rabi oscillation.

IV. CONCLUSIONS

The main idea we have been trying to promote is - the presence of two distinct resonances in the “Rabi Land scape” which is the plot of the generalized Rabi frequency versus the wave vector. The resonances are local minima in this plot. Unlike in graphene, where the conventional Rabi frequency has a form identical to those seen in two-level systems and semiconductors close to resonance - in topological insulator, this quantity shows a pronounced anisotropy in reciprocal space near the Dirac points. This may be attributed to a combination of the three dimensional nature of the Brillouin zone and the transverse nature of the electromagnetic field- a feature not shared by graphene since boththe graphene sheet and the plane electromagnetic wave being two dimensional, lead to an isotropic conventional Rabi frequency. In Dirac fermion systems, in addition to the conventional resonance located at the usual spot namely when the energy of the external photon matches the particle-hole energy, there is an anomalous resonance close to the Dirac points and far from conventional resonance. In surface states of topological insulators the location in wave vector space of this anomalous resonance is sometimes seen shifted relative to its location in graphene. Moreover, unlike ingraphene, where the anomalous Rabi frequency has a non-zero minimum value, in these systems, this minimum value vanishes making the modes of anomalous Rabi oscillations themselves ultra-relativistic.

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